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CONVECTIVE HEAT TRANSFER OF DIELECTRIC SUSPENSIONS IN COAXIAL CYLINDRICAL CHANNELS

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Abstract—The paper presents the results of a theoretical and experimental study of rheodynamics and convective heat transfer of weakly-conducting electrorheological suspensions in a horizontal coaxial cylindrical channel constituting a condenser. The solution of the problem has been verified using a non-linear viscoplastic model, postulated from electroviscosimetric experiments.

NOMENCLATURE

τ,	shear stress;
$\dot{\gamma}$,	shear rate gradient [s^{-1}];
ΔP,	static pressure drop [N/m^2];
λ_{eff}, a_{eff},	thermal conductivity and thermal diffusivity, respectively [W/m · deg] m^2/s;
q_1, q_2,	specific heat flux from internal and external wall of the channel, respectively [W/m^2];
α_0, α,	heat-transfer coefficient outside and inside the electric field [$W/m^2 \cdot deg$];
T_w, T_f,	temperature of the wall and of the fluid, respectively [deg];
n,	nonlinearity parameter;
τ_0,	initial shear stress (static yield stress) [N/m^2];
μ_p,	analogue of plastic viscosity [$N/s\, m^2$];
c,	solid phase concentration [wt.%];
L,	length of the channel;
r, x,	coefficients;
R_1, R_2, d,	inner and outer radii and diameter of the channel [m];
u,	flow velocity [m/s];
Q,	volumetric flow rate [cm^3/s];
q^*,	dimensionless flow rate;
a'_1, b'_1, b'_2, a'_2,	predicted coefficients;
E,	electric field intensity [kV/cm];
Pr,	Prandtl number.

FLUID disperse systems, sensitive to the external electric field, are finding increasing use in various areas of industry and technology. Their physicomechanical, electrophysical and thermophysical characteristics determine the valuable specific properties of the materials with assigned structure, obtainable with ever-wide use of electric fields, which makes it possible to substantially enhance efficiency and productiveness of technological processes and to improve the control of operational regimes of the equipment which employ fluid disperse media. There is a great number of inventions and patents advancing the use of the electric field effect on low-conducting disperse systems in the elements of automatic equipment (relays, travelling blocks, pressure and flow regulators), vibrators and vibrogenerators, dampers, sound amplifiers, recuperative heat exchangers, micromachines [1–4, 7]. In order to choose the optimum regimes of construction of such devices and of thermochemical apparatuses, and to determine the most effective heat-transfer conditions and characteristics of fluid disperse heat carriers, it is necessary to know in what way electric fields influence the processes of momentum and thermal energy transfer in such systems.

The present investigation has been undertaken with the aim of studying thermophysical and rheophysical properties, flow characteristics (velocity and stress profiles, the relation rate vs head), as well as the laws governing convective heat transfer in a coaxial cylindrical channel of a special type of dispersions—electrorheological suspensions. The properties of these suspensions experience a sharp change under the action of an electric field applied transversally to the direction of the shift. Thus, for example, effective viscosity increases by several orders up to its apparent solidification. This phenomenon, called the electrorheological effect, was discovered by Winslow [7], Klass and Martinek [5, 6] in 1949, and then studied experimentally by Luikov and Shulman *et al.* [8, 9]. Based on the above studies, three possible physical interpretations of the nature of the electrorheological effect have been suggested:

1. Competition between hydrodynamic and electrical orientation of polarized particles in the flow of suspensions.
2. Deformation of and interaction between diffuse shells, overlapping double electric layers surrounding the particles.
3. Oscillation of the particles and formation of solid phase structure in the gap between the electrodes.

The first two interpretations and their respective mathematical models are inadequate for predicting realistic characteristics of the shear flow and of the convective heat transfer of electrorheological suspensions. They underestimate increments in the effective viscosity by 2–3 orders as compared with the measured ones. At the Rheophysics Laboratory of the

Luikov Heat and Mass Transfer Institute, it has been demonstrated that it is the formation of the solid phase particles, i.e. formation of a vast number of bridge-like, dendrite, coagulated structures, that plays the role of a powerful physicochemical factor, capable of causing changes in internal friction, observed in the experiments. However, the dynamic nature of the structure formation process, intricate geometry of polydispersed structures, unknown potentials of interaction of particles with one another, with the electrodes and the external electric field, have not so far allowed formulation of a correct physicomathematical model of the electrorheological effect similar to the one used in the magnetic hydrodynamics or ferrofluids theory.

Therefore, in the present study we have used the technique of examining rheogrammes (curves of a uniaxial flow obtained from the electrorheometrical experiments) to allow for the effect which the inner changes in the structure of suspensions, induced by the electric field, exhibit on the transfer processes. Then, according to these rheogrammes, a corresponding rheological equation of state is selected, and the dependence of its coefficients on the solid phase content, activator concentration, electric field intensity and temperature is determined.

For the experiments we have chosen commonly used inexpensive mineral electrorheological suspensions of the following composition: solid phase—diatomite (0–5 wt%); carrier medium—transformer oil; activator—water.

Uniaxial flow curves are determined from the experiments on a specially fabricated cell of a coaxial cylindrical rotational viscosimeter, the ranges of electric intensity being 0–30 kW/cm, of rates of shear, 0.5–500 s^{-1}, and of temperatures, 293–363°C. In the absence of the electrical field, the rheogrammes of the chosen suspensions are linear (Fig. 1). Their inclination is comparatively small, and the plastic component of the flow is insignificant. With the electrical field applied, the flow curves undergo the following changes: first, at small and moderate rates of shear a non-linear segment appears, second, with increasing electric intensity, the curves shift upwards along the axis of intensity, i.e. the yield stress and effective viscosity are increased. Thus, application of a static electric field, transversal to the shift surfaces, and in particular of a low-divergent radial electric field, essentially influences the character of the flow. The limiting shear stress, τ_0, as well as the length and curvature of the non-linear segment of the rheogrammes, increase considerably faster than in direct proportion to the electric field intensity. The obtained flow curves are typical of nonlinear viscoplastic media and are adequately described by the generalized equation of rheological state [11]

$$\tau^{1/n} = \tau_0^{1/n} + (\mu_p \dot{\gamma})^{1/n}, \quad n\text{—of any value.} \qquad (1)$$

In Figs. 2–4, the plasticity factor, τ_0, nonlinearity parameter, n, and plastic viscosity analogue, μ_p, are plotted vs the electric intensity. The analysis makes it possible for calculations to assume the plastic viscosity analogue constant (μ_p = const), and to suggest a linear dependence on the electric intensity for the parameter n, and a quadratic one for the shear stress,

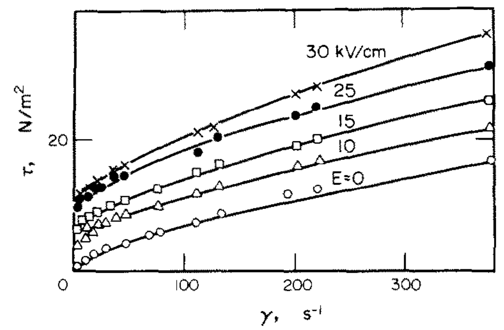
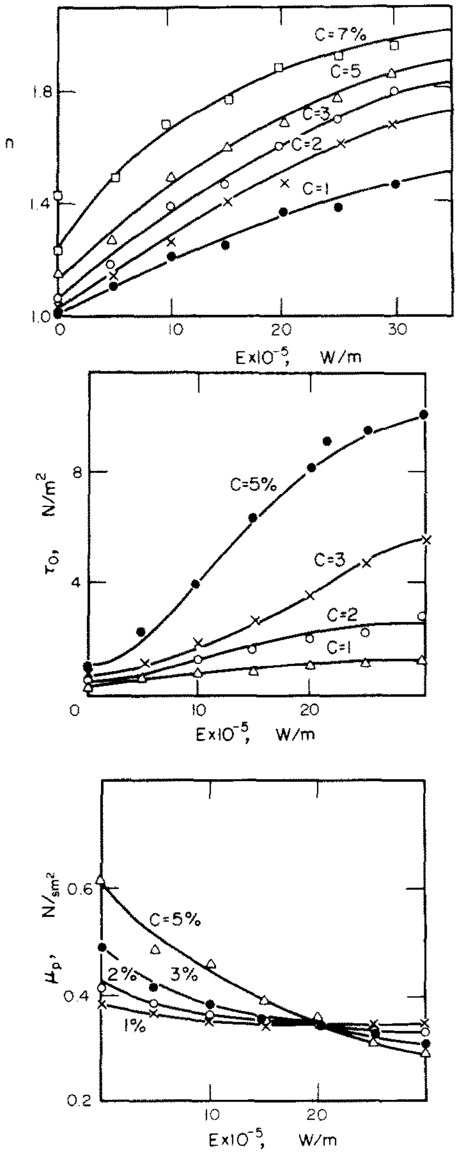


FIG. 1. Flow curves for the 5% suspension of diatomite in transformer oil with the electric field applied.



FIGS. 2–4. Dependence of the rheological parameters of electrorheological suspensions (initial shear stress, nonlinearity parameter and analogue of plastic viscosity) on the intensity of the applied electric field.

τ_0. This, then, allows prediction of the flow and heat-transfer characteristics of low-conducting electrorheological suspensions in the gap of the coaxial cylindrical channel of the condenser. Now, let us consider a dynamic problem of uniaxial stabilized shear flow. The radial shear stress profile in the cross-section of the channel is of the form

$$\tau = \tfrac{1}{2}R_2\frac{\Delta P}{L}\left(\frac{r}{R_2} - \frac{\bar{\lambda}^2 R_2}{r}\right) \tag{2}$$

where $\bar{\lambda}^2$ is a certain constant, describing the position of the zero stress isoline, $\tau = 0$, at $r = \bar{\lambda}R_2$, in the channel. The total flow region is subdivided into 3 zones: a quasisolid core and 2 wall regions (Fig. 5). Then,

$$\tau^{1/n} = \tau_0^{1/n} + \left(\mu_p \frac{\mathrm{d}u_1}{\mathrm{d}r}\right)^{1/n};$$

$$R_1 < r < r_1,\ \tau > 0 \ \ \text{1st zone}$$

$$(-\tau)^{1/n} = \tau_0^{1/n} + \left(-\mu_p \frac{\mathrm{d}u_2}{\mathrm{d}r}\right)^{1/n};$$

$$r_2 < r < R_2,\ \tau < 0 \ \ \text{2nd zone} \tag{3}$$

$$\tau = \tau_0. \quad \text{3rd zone}$$

The boundary conditions for the flow clinging to the channel walls are

$$u_1(r) = 0, \qquad r = R_1 \tag{4}$$

$$u_2(r) = 0, \qquad r = R_2. \tag{5}$$

Introduction of the dimensionless variables

$$\rho = r/R_2, \qquad \phi = \frac{2\mu_p L}{R_2^2 \Delta P}u,$$

$$\beta_0 = \frac{2\tau_0 L}{R_2 \Delta P}, \qquad \kappa = \frac{R_1}{R_2}$$

and direct integration of the system (2)–(3) yields the following expressions for the flow velocities in the 1st and 2nd zones

$$\begin{aligned}\phi_1 &= \bar{\lambda}^2 \ln \rho/\kappa - \tfrac{1}{2}(\rho^2 - \kappa^2) + \sum_{p=1}^{\infty} (-1)^p C_n^p \beta_0^{p/n} \bar{\lambda}^{2(1-p/n)} \cdot (\alpha_1 + \alpha_2)\\ \phi_2 &= \bar{\lambda}^2 \ln \rho + \tfrac{1}{2}(1 - \rho^2) + \sum_{p=1}^{\infty} (-1)^p C_n^p \beta_0^{p/n}(\beta_1 + \beta_2)\end{aligned} \tag{6}$$

where

$$\alpha_1 = \frac{\rho^{p/n} - \kappa^{p/n}}{p/n}, \quad \alpha_2 = \sum_{i=1}^{\infty} \frac{(-1)^i C_{1-p/n}^i \cdot \bar{\lambda}^{2i}(\rho^{2i+p/n} - \kappa^{2i+p/n})}{2i + p/n}$$

$$\beta_1 = \begin{cases} \dfrac{1-\rho^{2-p/n}}{2-p/n}; & p/n \neq 2\\ \ln \rho; & p/n = 2 \end{cases} \qquad \beta_2 = \sum_{i=1}^{\infty} (-1)^i C_{1-p/n}^i \cdot \bar{\lambda}^{2i} \begin{cases} \dfrac{1-\rho^{2-p/n-2i}}{2-p/n-2i}; & \dfrac{p}{n} \neq 2(1-i)\\ -\ln \rho; & p/n = 2(1-i) \end{cases}$$

$$C_{1-p/n}^i = \frac{(1-p/n)\ldots(1-p/n-i+1)}{i!}.$$

The boundaries of the quasisolid core, ρ_1, ρ_2, and the integration constant, $\bar{\lambda}$, are determined based on:

1. The kinematic condition of equal velocities on the boundaries of the quasisolid core of the flow, $\phi_1(\rho_1) = \phi_2(\rho_2)$;

2. The dynamic condition of equal forces acting on the core, $\rho_2 - \rho_1 = \beta_0$;

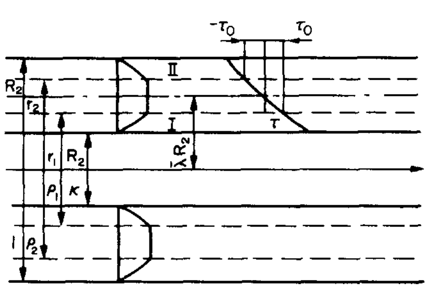


FIG. 5. Velocity profile of a nonlinear viscoplastic medium in a coaxial cylindrical channel.

3. The condition of the shear stress on the boundaries of the core being equal to the yield stress, resulting in the relation $\bar{\lambda}^2 = \rho_1 \cdot \rho_2$.

The obtained transcendental equation was solved numerically for $n = 1$; 1.1; 1.2; ... 3 on an electronic computer "Minsk-22". As a result, the effect of the rheological parameters, n, $\tau_0(\beta_0)$, on the velocity of the quasisolid core and dimensionless flow rate q^*, has been determined. It has turned out that with increase in τ_0, the values of ϕ_0 and q^* decrease continuously, this decrease being the faster, the larger was the nonlinearity parameter. With increasing rheological parameters n, τ_0, the velocity profile, obtained for constant volumetric flow rates, becomes less convex and more filled, while the width of the quasisolid core and the velocity gradients in the wall region increase (Fig. 6). An unambiguous dependence of the parameters τ_0 and n on the electric field intensity makes it possible to establish the relationship between the relative volumetric flow rate and the pressure drop for different values of E, described by the formula

$$Q/Q_0 = 1 - \frac{a_1' + a_2' E^2}{b_1' + b_2' \Delta P}$$

whose coefficients are calculated by the Gauss–Seidel method.

The predicted results have been compared with the experiments, in which the volumetric flow rate, Q, and pressure drop, ΔP, of the developed flow of electrorheological suspensions in the working section of the set-up, have been measured. The working section was a coaxial cylindrical gap 500 mm long. The

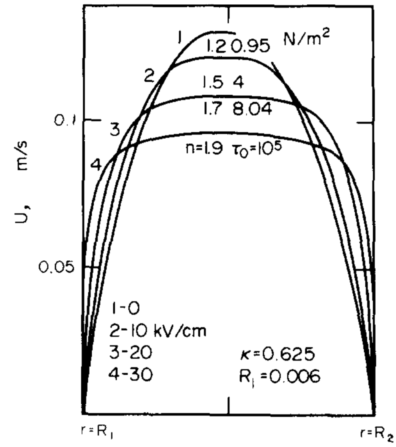


FIG. 6. Predicted velocity profile of the electrorheological suspension in a horizontal coaxial cylindrical channel-condenser.

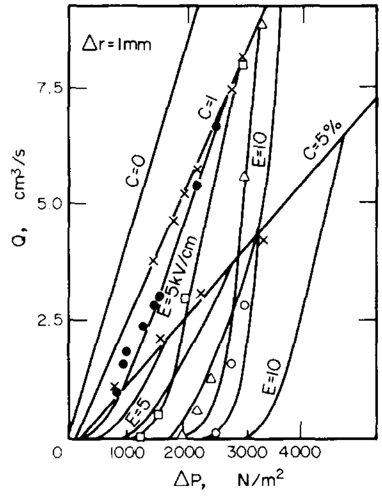


FIG. 7. Dependence of the volumetric flow rate of the electrorheological suspension on the pressure drop in a coaxial cylindrical channel-condenser for different solid phase (diatomite) concentrations and on electric field intensity:—theory, ●, □, ×, ▲, △, ○—experiment.

external pipe had a constant inner diameter of 16 mm, while the internal cylindrical rod was 12, 14 or 10.7 mm in diameter. With the external pipe earthed, a controlled high electrical voltage was applied to the internal rod. Hydrostatic head was created by a system of recasting tanks. As expected, under no-field conditions the relation rate vs head is linear (Fig. 7). The applied electric field gives rise to a parabolic relation of the type $Q^n \sim \Delta P$, till the curves reach the points, where the head shear stresses considerably exceed the retarding electrical effect. Measurements have been made for different solid phase concentrations and varied temperatures, heads and flow sections of the channel. The agreement between theoretical and experimental results within the limits of the maximum deviations constituted approximately 11%, which proves the validity of the approximate approach based on the rheogrammes. Knowledge of the velocity profile in the channel (equation 6) allows solution of the problem of convective heat transfer of a steady head electrorheological suspensions flow in a horizontal coaxial cylindrical channel-condenser at the boundary conditions of the second kind on the wall (q_1 = constant, $q_2 = 0$), which is reduced to solution of the thermal energy transfer equation in the form

$$u(r)\frac{\partial T}{\partial x} = a_{\mathrm{eff}}\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right). \qquad (7)$$

Let us use the following assumptions: the heat transfer is developed, physical parameters of the medium are independent of temperature; dissipation of mechanical energy, radiant heat transfer and longitudinal heat conduction are neglected, there are no internal heat sources and sinks, the fluid flow is laminar, clinging to the channel walls. The boundary conditions are

$$\begin{aligned} -\lambda_{\mathrm{eff}}\frac{\partial T}{\partial r} &= q_1, \quad r = R_1 \\ \frac{\partial T}{\partial r} &= 0, \quad r = R_2 \qquad (8) \\ T &= T_0, \quad x = x_0. \end{aligned}$$

$\lambda_{\mathrm{eff}}(E)$, $a_{\mathrm{eff}}(E)$ are the effective values of thermophysical coefficients, obtained in special experiments for different quantities using non-stationary techniques.

As is clear from Fig. 8, the electric field, which changes considerably the structure of electrorheological suspensions, influences effective thermal

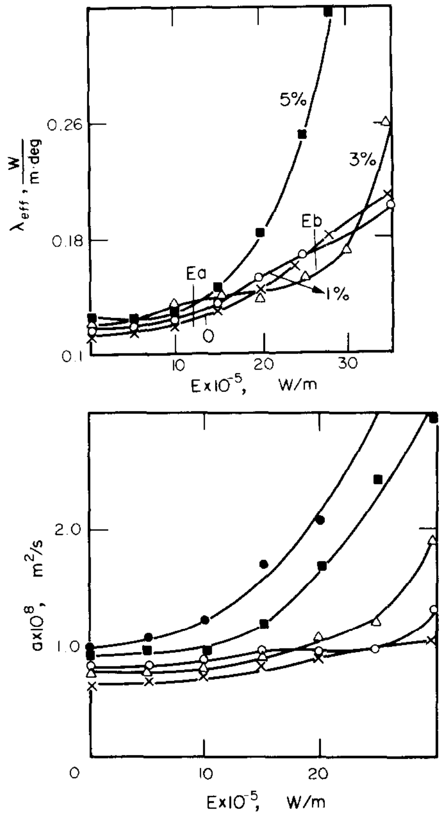


FIG. 8. Dependence of thermophysical characteristics: (a) effective thermal conductivity coefficient; (b) effective thermal diffusivity coefficient, on the electronic field intensity.

conductivity and diffusivity coefficients. While thermal conductivity of clean transformer oil increases only slightly with electric intensity, E, because of electric convection, this process is further intensified by adding to the oil of solid phase particles. Thus, for $C = 5\%$ at $E \simeq 24\,\text{kV/cm}$, maximum change in the value of λ_{eff} was 80%. The increase in λ_{eff} and a_{eff} with electric field intensity is connected with the increase of the conductive component of the overall heat transfer through the contact spots between the solid particles, and with intensification of electric convection in the spaces between the dispersed particles.

For equations (7)–(8), in such statement as above, strict analytical solutions are possible only if the velocity distribution is described by simple relations. Therefore, for this purpose numerical methods have been used, like running on an electronic computer "Minsk-22". The results are presented in Fig. 9.

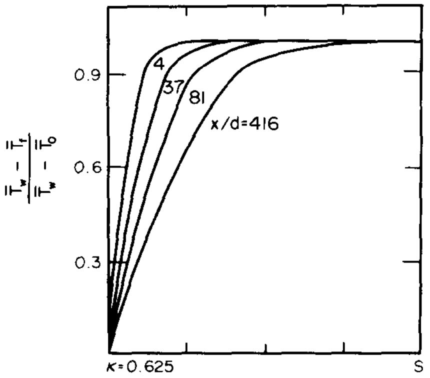


Fig. 9. Dimensionless temperature profile of the medium (1) in a coaxial cylindrical channel for different values of the reduced length x/d.

A marked change is observed in the dimensionless temperature drops both across the channel and along its axis. The temperature gradient on the internal surface remained practically the same, since the axial rod was ohmically heated. Convective heat transfer in the electrorheological suspensions at the chosen conditions occurs in the narrow wall (external) region, where maximum nonuniformities of the temperature field are observed. This region is followed by an isothermal heat core, whose temperature is equal to the inlet temperature, T_0. The thickness of the thermal boundary layer depends on the reduced length of the channel, thermophysical properties of the medium and most strongly on the hydrodynamic flow parameters. For the considered highly viscous media, $Pr \gg 1$. Heat-transfer intensity under such conditions is determined mainly by the velocity gradient in the wall region, i.e. by the form-factor $\beta^* = (\partial u/\partial r)_{r=R_1}$. The calculations made for non-linear viscoplastic media, described by the model (1), have proved that, at a fixed flow discharge of the suspension through the channel cross-section, intensification of the electrorheological effect leads to increase of the velocity gradient in the wall region, i.e. just in the region of the thermal boundary layer. Thus, the theoretical solution of the problem shows that with increasing electric intensity, heat transfer should be enhanced. This conclusion is confirmed by the experimental data, obtained on the above-described set-up at heat fluxes on the inner rod being 15, 93, 5 and $56\,\text{kW/m}^2$, and volumetric flow rates of suspension, Q, being 2.5–$25\,\text{cm}^3/\text{s}$. A detailed schematic diagram of the set-up is given in [13].

The experimental procedure was as follows. Suspension, heated uniformly up to the required temperature, was supplied to the heated working contour from the upper head tank. Its flow velocity (volumetric flow rate) was controlled by changing the height of position of the discharge tank. After the development of the steady-state heat-transfer electric voltage was applied to the electrodes, and in 5 min (required for establishing a new heat-transfer regime) the outputs of thermocouples and resistance thermometers were read. By the amount of heat supplied and the temperature head, convective heat-transfer coefficient was determined $\alpha = q_1/(\bar{T}_w - \bar{T})$. Figure 10 presents the relationship of the relative coefficient, $\alpha/\alpha_0(E)$, for different solid phase concentrations. Three degrees in the process development can be distinguished. A noticeable increase in the heat-transfer intensity starts to exhibit itself only after the electric intensity has reached a certain value, namely E threshold ~ 5–$7\,\text{kV/cm}$. Then follows the area of considerable intensification, and, finally, at the electric field intensities of the order of $25\,\text{kV/cm}$, saturation is observed. The experimentally derived relation, $\alpha/\alpha_0(E)$, qualitatively and quantitatively (the scatter not exceeding 18%) agrees with the theoretical calculation of the stated heat-transfer problem. This justifies a rheophysical approach to the problems of dynamics and convective heat transfer of non-linear viscoplastic media, in particular, of low-conducting electrorheological suspensions flowing in a horizontal coaxial cylindrical channel-condenser.

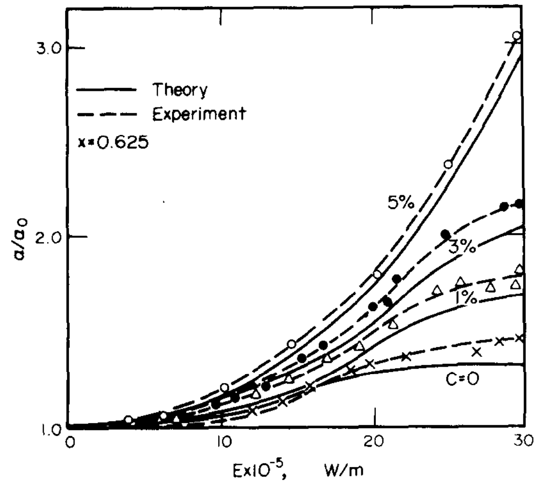


Fig. 10. Dependence of the relative heat-transfer coefficient of diatomite suspensions in transformer oil on the electric field intensity in a laminar flow in a horizontal coaxial cylindrical channel-condenser.

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TRANSFERT THERMIQUE PAR CONVECTION A DES SUSPENSIONS DIELECTRIQUES DANS DES CANAUX ENTRE CYLINDRES COAXIAUX

Résumé—L'article présente les résultats d'une étude théorique et expérimentale du transfert thermique convectif de suspensions électrorhéologiques faiblement conductrices, dans un canal horizontal entre cylindres coaxiaux et constituant un condensateur. La solution du problème a été vérifié par un modèle viscoplastique et non linéaire, établi à partir des expériences électroviscosimétriques.

KONVEKTIVER WÄRMEAUSTAUSCH VON DIELEKTRISCHEN AUFHÄNGUNGEN IN ZYLINDRISCHEN KOAXIALEN KANÄLEN

Zusammenfassung—Die Arbeit zeigt die Ergebnisse einer theoretischen und experimentellen Studie über rheodynamischen und konvektiven Wärmeaustausch von schwach leitenden elektrorheologischen Aufhängungen in einem horizontalen zylindrisch-koaxialen Kanal, der einen Kondensator bildet. Die Lösung des Problems wurde anhand eines nicht-linearen viskoplastischen Modells, aufgestellt für elektroviskosimetrische Versuche, bestätigt.

КОНВЕКТИВНЫЙ ТЕПЛООБМЕН ДИЭЛЕКТРИЧЕСКИХ СУСПЕНЗИЙ В СООСНОЦИЛИНДРИЧЕСКИХ КАНАЛАХ

Аннотация — Приведены результаты теоретико-экспериментального исследования реодинамики и конвективного теплообмена слабопроводящих электрореологических суспензий в горизонтальном сооснοцилиндрическом канале-конденсаторе. Решение задачи проверено на основе нелинейно вязкопластической модели, полученной из электровискозометрических опытов.